NAME______________________________ DATE __________

Study Guide

Student Edition
Pages 514–521

Polygons

A **polygon** is a plane figure formed by a finite number of segments such that (1) sides that have a common endpoint are noncollinear and (2) each side intersects exactly two other sides, but only at their endpoints. A **convex polygon** is a polygon such that no line containing a side of the polygon contains a point in the interior of the polygon. Convex polygons with all sides congruent and all angles congruent are called **regular**.

The following two theorems involve the interior and exterior angles of a convex polygon.

Interior Angle Sum Theorem	If a convex polygon has n sides and S is the sum of the measures of its interior angles, then $S = 180(n - 2)$.
Exterior Angle Sum Theorem	If a polygon is convex, then the sum of the measures of the exterior angles, one at each vertex, is 360.

Example: Find the sum of the measures of the interior angles of a convex polygon with 13 sides.

$S = 180(n - 2)$ **Interior Angle Sum Theorem**
$S = 180(13 - 2)$
$S = 180(11)$
$S = 1980$

Find the sum of the measures of the interior angles of each convex polygon.

1. 10-gon **2.** 16-gon **3.** 30-gon

The measure of an exterior angle of a regular polygon is given. Find the number of sides of the polygon.

4. 30 **5.** 20 **6.** 5

The number of sides of a regular polygon is given. Find the measures of an interior angle and an exterior angle for each polygon.

7. 18 **8.** 36 **9.** 25

10. The measure of the interior angle of a regular polygon is 157.5. Find the number of sides of the polygon.

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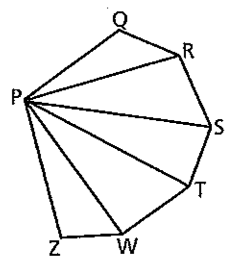
The Sum of All Interior Angles

The angles inside a polygon are called **interior angles.**

To find the sum of the measures of the interior angles:
180° (n – 2) = sum of interior angles
n = the number of sides in the polygon.



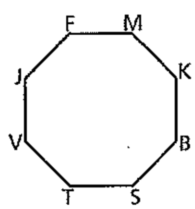
This heptagon has 7 sides. The diagonals divide the figure into 5 different triangles, each totaling 180°.

180° (7 – 2) = sum of interior angles
180° (5) = 900°

The sum of the interior angles in this heptagon is 900°.

For each polygon, mark one set of diagonals. Then use the formula to find the sum of the interior angles.

1



180° (n – 2) = sum of interior angles

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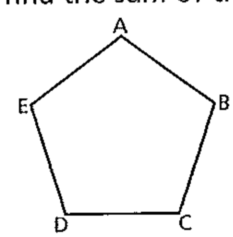
sum of interior angles = ____________

2



180° (n – 2) = sum of interior angles

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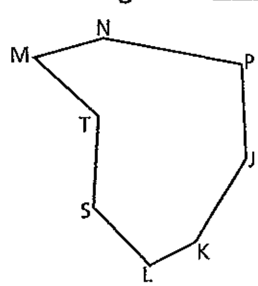
sum of interior angles = ____________

3



180° (n – 2) = sum of interior angles

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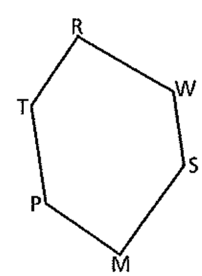
____

sum of interior angles = ____________

4



180° (n – 2) = sum of interior angles

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sum of interior angles = ____________

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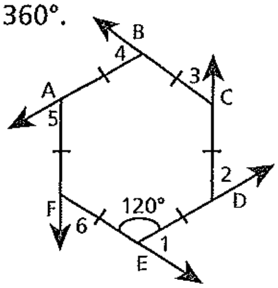
The Sum of All Exterior Angles

Exterior angles are outside a polygon. An exterior angle is formed by a side of a polygon and an extension of an adjacent side.

The sum of the exterior angles for a polygon is 360°.



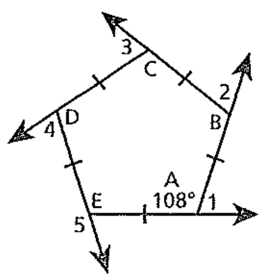
The interior and exterior angles at each vertex form a linear pair.
If the interior angle is known, the exterior angle must be its supplement.

$\angle FED + \angle 1 = 180°$
If $\angle FED = 120°$, then $\angle 1 = 60°$.

$60° \times 6$ exterior angles = 360°

For each polygon, find the linear pair of the marked angles. Prove that the sum of the angles equals 360°.

1



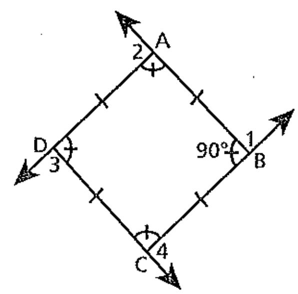
m∠ ________ + m∠ ________ = 180°

m∠ ________ = ________

m∠ ________ = ________

________ × ________ angles = 360°

2



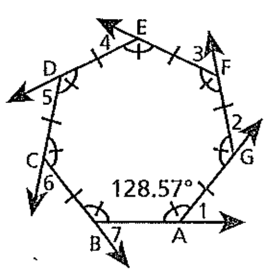
m∠ ________ + m∠ ________ = 180°

m∠ ________ = ________

m∠ ________ = ________

________ × ________ angles = 360°

3



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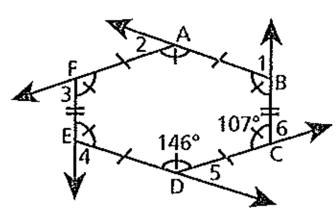
________ × ________ angles ≈ 360°

4



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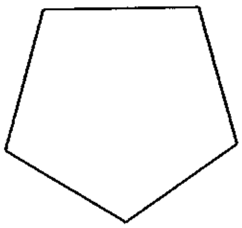
Interior Angle Sum of Convex Polygons

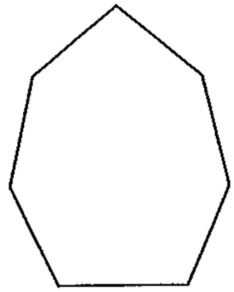
To find the total sum of the angles inside a convex polygon, divide the polygon into triangles by drawing all diagonals from one vertex. Count the number of triangles and multiply by ____°. (Hint: how many degrees are in a triangle?)

1.



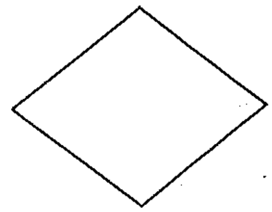
number of Δs ________
interior $\angle$ sum ________

2.



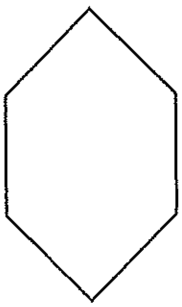
number of Δs ________
interior $\angle$ sum ________

3.



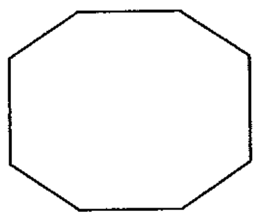
number of Δs ________
interior $\angle$ sum ________

4.



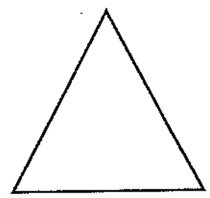
number of Δs ________
interior $\angle$ sum ________

5.



number of Δs ________
interior $\angle$ sum ________

6.



number of Δs ________
interior $\angle$ sum ________

Look for a pattern above. Compare the number of sides in the polygon to the number of triangles created. Use this pattern to determine the following:

7. interior $\angle$ sum of a decagon

8. interior $\angle$ sum of a dodecagon

9. interior $\angle$ sum of a 32–sided polygon

Angles in Regular Polygons

Follow these rules to find measures of angles in regular polygons:

number of triangles	=	number of sides – 2
interior angle sum	=	number of Δs • 180°
one interior angle	=	interior angle sum ÷ number of sides
exterior angle sum	=	360°
one exterior angle	=	360° ÷ number of sides

Solve for the indicated measure. Locate answers in the decoder to find the name of the man who patented the geodesic dome (a structure like the one at EPCOT center in Orlando, Florida). ALL POLYGONS REFERRED TO BELOW ARE REGULAR.

1. The measure of one interior angle of a hexagon.
2. The measure of one interior angle of a dodecagon.
3. The measure of one exterior angle of an octagon.
4. The measure of one exterior angle of a quadrilateral.
5. The interior angle sum of a pentagon.
6. The exterior angle sum of a 27–sided polygon.
7. The measure of one interior angle of a nonagon.
8. The measure of one exterior angle of a decagon.
9. The measure of one exterior angle of a dodecagon.
10. The measure of one interior angle of a triangle.
11. The measure of one interior angle of a pentagon.
12. The measure of one exterior angle of a nonagon.
13. The measure of one interior angle of an octagon.

decoder	
360°	B
40°	C
140°	E
30°	F
135°	I
60°	K
120°	L
150°	M
108°	N
90°	R
36°	S
45°	T
540°	U

____ . ____ ____ ____ ____ ____ ____ ____ ____ ____ ____ ____
4 6 5 12 10 2 13 11 8 3 7 4

____ ____ ____ ____ ____ ____
9 5 1 1 7 4